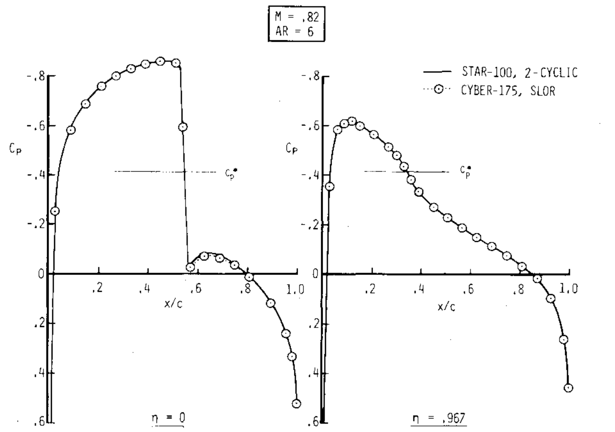

Fig. 1 Pressure distributions on a rectangular wing at $M_\infty = .82$, $\alpha = 0$ deg.

approach proved to be very satisfactory and some preliminary results are presented in the next section.

A short explanation of the tri-diagonal solution in vector mode is in order. Solving a single set of tri-diagonal equations is inherently a recursive operation and thus cannot be vectorized in any reasonable manner. In this case, since all the equations for half the y-z plane were derived simultaneously, one has the prospect of solving them in parallel. In this case, one forms vector operations of length equal to the number of sets of tri-diagonal equations. For the STAR-100 this is a very short vector ($M/2$) but it is considerably better than running in scalar mode.

III. Results

A computer code was developed for the STAR-100. All the basic operations used in the SLOR are retained, including the time consuming process of determining the local flow character, i.e., subsonic, sonic, supersonic, or shock. The code for the 2-cyclic approach has only 5% in common with the first version in which column relaxation was carried out simultaneously for all the columns in an x-plane. This is interesting because their simulations on the scalar machine differ only by one simple DO loop. This experience tells us that numerical research for the vector machine really ought to be simulated on scalar machine first. The well-matured software of scalar machines makes it much easier to change the direction of approach.

The code was used for a rectangular wing of $AR = 6$ with constant NACA 0012 sections.[6] A mesh of $64 \times 28 \times 20$ was used. The pressure distributions at $M_\infty = .82$, $\alpha = 0$ deg is shown in Fig. 1 for the wing root and the wing tip. It may be seen that there were slight differences between the results obtained from the SLOR (CDC CYBER 175) and 2-cyclic approach (CDC STAR-100), especially behind the shock. When the 2-cyclic approach was carried out on the CDC CYBER 175, the same character was observed. It is concluded that the variation of the pressure distribution was a result of the 2-cyclic approach, not that of using the STAR-100 computer. The computer time used for each case is shown in Table 1. The numbers of iterations required to reach the same level of convergence are nearly identical for SLOR and 2-cyclic approach. Hence, the high speed of operation on STAR-100 was not offset by adverse effects from the algorithm change. A speed increase factor of (3.4) was achieved. It is believed that a much better gain may be achieved if longer vectors could be used.

Table 1 Computer time used for SLOR and 2-cyclic technique

	SLOR (CYBER-175)	2-cyclic (STAR-100)
CPU time, s	261.1	76.5
No. of iterations to reach convergence	266	265

IV. Conclusions

From the exercise carried out at Boeing, we discovered that in order to use efficiently a vector machine such as STAR-100, the function of the code should be tested first on a scalar machine. This is mainly because of the mature software for scalar machine and its great flexibility to allow the user to modify program functions. A major rewrite (95% of the code) had to be made when a simple switch from SLOR to 2-cyclic approach became necessary on the STAR-100. The development of an explicit scheme such as the one by Keller and Jameson[5] does not seem to be necessary, especially when the explicit scheme converged three times slower than the implicit SLOR even for a 2-D problem. On the other hand, the short and moderately long vectors ($M = 28$ and $M \times N = 560$) used in this study obviously paid some penalty for start-up time. Further studies of such problems as how to increase the vector length are necessary to fully take advantage of the vector computer such as the STAR-100.

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Conservation Errors in Axisymmetric Finite-Difference Equations

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I. Introduction

THE prediction of many flows of practical interest requires that the solution be carried out numerically. The governing partial-differential equations are replaced by finite-difference equations. The errors that can arise when such a procedure is followed are usually attributed to numerical

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inaccuracies: fineness of the finite-difference grid, roundoff errors in the computer, etc. The purpose of this Note is to discuss an additional type of error that can arise in the solution: the inability of the finite-difference equations to preserve the global conservation properties of the flow.

The problem of conservation has been discussed by several investigators and is summarized by Roache.[1] Fromm[2] showed the desirability of employing a conservative scheme. Arakawa[3] developed a scheme for two-dimensional flow that conserved vorticity, mean kinetic energy, and mean square vorticity. Torrance[4] compared various finite-difference methods, both conservative and nonconservative. Parmentier and Torrance[5] developed consistent finite-difference approximations for use in curvilinear coordinates.

All of these previous works have dealt with the finite-difference representation of the governing differential equations expressed in the geometric variables x, r. However, in many problems, it is convenient to perform the calculations in a transformed coordinate system, with the stream function ξ replacing the radial position.[6,7] In this case, different conservation problems arise at the centerline of the flow.

II. Analysis

The governing equations for conservation of momentum, energy, or species mass in axisymmetric flow can be reduced to a set of equations of the form:

$$\frac{\partial F}{\partial x} = \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(a\frac{\partial F}{\partial \xi}\right) + P \tag{1}$$

where F is the quantity of interest (velocity, temperature, etc.), x is the axial distance, and P is a production term. The stream function ξ is defined as

$$\xi\frac{\partial \xi}{\partial r} \equiv \rho u r \qquad \xi\frac{\partial \xi}{\partial x} \equiv -\rho v r \tag{2}$$

satisfying conservation of total mass identically. The parameter a is

$$a \equiv \rho u r^2 \mu / \sigma_F \xi \tag{3}$$

where μ is the viscosity and σ_F is an appropriate Prandtl number, which will be taken as unity.

The diffusion term in Eq. (1), when integrated over the cross section of the flow, is zero or a constant for unbounded or bounded flows, respectively. Without loss of generality, we can investigate the former case. To insure no spurious sources or sinks in the numerical computation, the finite-difference analog of this integration must also be zero:

$$\sum_{n=0}^{N}\left\{\frac{1}{\xi}\frac{\partial}{\partial \xi}\left(\alpha\frac{\partial F}{\partial \xi}\right)\right\}_n \xi_n \Delta\xi = 0 \tag{4}$$

The notation $\{\ \}_n$ refers to the particular finite-difference scheme chosen.

In many finite-difference formulations of the governing equations, the centerline of the flow is treated differently than the remaining grid points since the diffusion term becomes indeterminate at $\xi = 0$. In that case, it is necessary to rewrite Eq. (4) as a centerline contribution plus a summation:

$$\sum_{n=0}^{N} f_n = f_0 + \sum_{n=1}^{N} f_n \tag{5}$$

Several types of finite-difference approximations can be used. In most cases[6,7] the diffusion term is approximated by an expression of the form:

$$\frac{\partial}{\partial \xi}\left(a\frac{\partial F}{\partial \xi}\right) \cong \frac{a_+}{\Delta\xi}\frac{\Delta F}{\Delta \xi}_+ - \frac{a_-}{\Delta\xi}\frac{\Delta F}{\Delta \xi}_- = \frac{1}{2\Delta\xi^2}$$
$$\times\{(a_n + a_{n+1})(F_{n+1} - F_n) - (a_n + a_{n-1})(F_n - F_{n-1})\} \tag{6}$$

at locations removed from the centerline. In contrast with other formulations in which the diffusion term is differentiated prior to finite differencing:[8]

$$\frac{\partial}{\partial \xi}\left(a\frac{\partial F}{\partial \xi}\right) \cong a\frac{\Delta^2 F}{\Delta \xi^2} + \frac{\Delta F}{\Delta \xi}\frac{\Delta a}{\Delta \xi} \tag{7}$$

Eq. (6) preserves the conservative nature of the equations. It can be readily shown that Eq. (7) violates the conservation principle.

Substituting Eq. (6) into Eq. (5) and performing the indicated sum gives:

$$\sum_{n=0}^{N} f_n = f_0 - \frac{a_0 + a_1}{2\Delta\xi} 011 - F_0) \tag{8}$$

To determine whether conservation is achieved we must add the appropriate centerline value to the right-hand side of Eq. (8). If a limiting form of the conservation equation is used at the centerline of the flow, the diffusion term reduces to

$$\lim_{\xi \to 0} \frac{1}{\xi}\frac{\partial}{\partial \xi}\left(a\frac{\partial F}{\partial \xi}\right) = 2\mu_0\frac{\partial^2 F}{\partial \xi^2} \tag{9}$$

The numerical computation at the centerline can be accomplished in two ways. First, the location $\xi = 0$ can be treated as a true singularity.[6] In this case, the contribution of the centerline to the summation is:

$$\left\{2\mu_0\frac{\partial^2 F}{\partial \xi^2}\right\}_0 \xi_0 \Delta\xi = 0 \tag{10}$$

since $\xi = 0$ at the centerline. Hence, conservation is not achieved.

A second way to treat the centerline is to assume that the finite-difference equations, although written for the centerline, apply to a finite-sized streamtube with nonzero mass flow. Since the term $\xi\Delta\xi$ represents the mass flow in each streamtube, its value for the center tube is $\Delta\xi^2/4$. The centerline contribution to Eq. (8) is:

$$f_0 = \mu_0 (F_1 - F_0) \tag{11}$$

In this case as well conservation is not, in general, achieved since

$$\mu_0 \neq (a_0 + a_1)/2\Delta\xi \tag{12}$$

except when the viscosity is independent of position and the product of density times velocity is constant.

The conservation difficulty can be circumvented by not employing a limiting form of the equations at the centerline. The finite-difference scheme of Eq. (6) is obtained by integrating the defining differential equations over the grid points, from $(\xi - \Delta\xi/2)$ to $(\xi + \Delta\xi/2)$. The same procedure can be used at the centerline where the integration limits are now 0 and $\Delta\xi/2$. The result is:

$$f_0 = (a_{1/2}/\Delta\xi)(F_1 - F_0) \tag{13}$$

Conservation is then achieved provided $a_{1/2}$ is defined as the average of a_0 and a_1.

III. Conclusions

A consistent set of finite-difference equations for the numerical simulation of problems in heat, mass, and/or momentum transfer requires that the finite-difference set possess the same conservation properties as the original differential equations. Not all finite-difference formulations demonstrate this conservation. In particular, the use of a limiting form of the governing equations at the centerline will not result in overall conservation for any finite-difference formulation. Expanding the diffusive term as the sum of two derivatives also leads to errors, regardless of whether or not the centerline of the flow is treated in a limiting fashion.

In order to arrive at a consistent form of the finite-difference equations, the governing differential equation should be integrated over the grid space. This applies to the centerline of the flowfield as well as the exterior points.

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Characteristics of the Split Film Sensor

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Introduction

A recent innovation in hot-film anemometry has been the development of the split film sensor.[1] The sensor consists of two electrically independent films on a single quartz fiber, as shown in Fig. 1. If each film is operated with a separate anemometer circuit, the total heat transfer from both films provides a measure of the magnitude of the velocity vector normal to the axis of the fiber, U_N, while the difference in the heat transfer from the two films provides a measure of the component of the velocity vector perpendicular to the plane of the splits, i.e., $U_N \sin\theta$. The use of the split film sensor has been dependent on data from experimental calibration. This Note presents results obtained from a numerical simulation which shows excellent agreement with experimental data for steady flows. Some effects arising from unsteadiness in the flow are discussed and an analysis of some sources of error in the use of the split film sensor is presented.

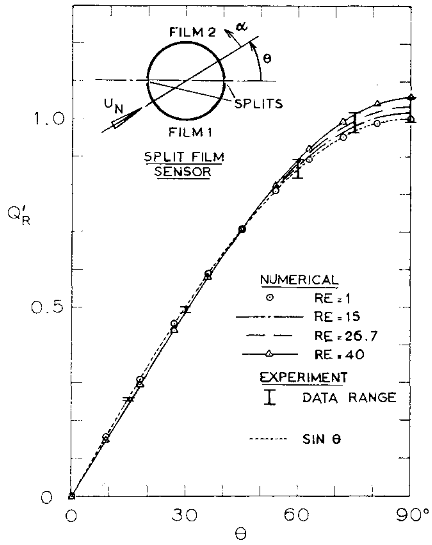

Fig. 1 Difference in heat transfer from the two films of a split film sensor in steady flow; theory and experiment. Values are scaled to make all curves pass through sin θ at $\theta = 45$ deg.

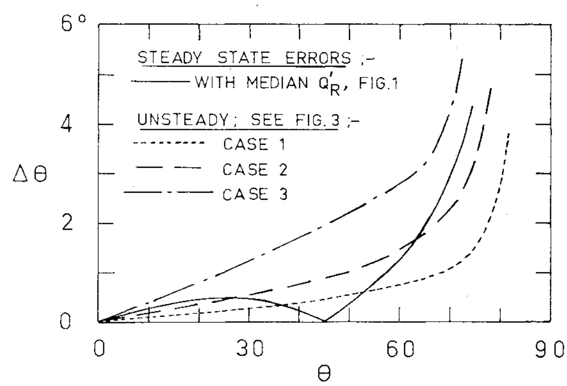

Fig. 2 Errors associated with use of single calibration curve for split film sensor in steady and unsteady flows.

Numerical Simulation

The heat transfer from a heated circular cylinder in crossflow was calculated by numerical integration of the Navier-Stokes equation, coupled with the energy equation. The flow was two-dimensional and the fluid was assumed to be imcompressible and to have properties which are invariant with temperature.

The heat-transfer characteristics were computed for steady flows at values of *Re,* the Reynolds number based on cylinder diameter d ranging from 1 to 40 and for a number of unsteady flows. In all calculations the value used for the Prandtl number was 0.7. The details of the numerical simulation are given by Apelt and Ledwich.[2] The values obtained for the total heat transfer from the cylinder in steady crossflows are in very close agreement with those obtained in a different numerical simulation reported by Dennis et al.[3] and are larger than the experimental results of Collis and Williams[4] by

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